Blacktown Boys' High School

2022 Year 12

HSC Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for this paper
- All diagrams are not drawn to scale
- In Questions in Section II, show all relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 3 – 10)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 11 – 40)

- Attempt Questions 11 – 32
- Allow about 2 hours and 45 minutes for this section

Assessor: K. Villanueva & X. Chirgwin

Student Name:_____

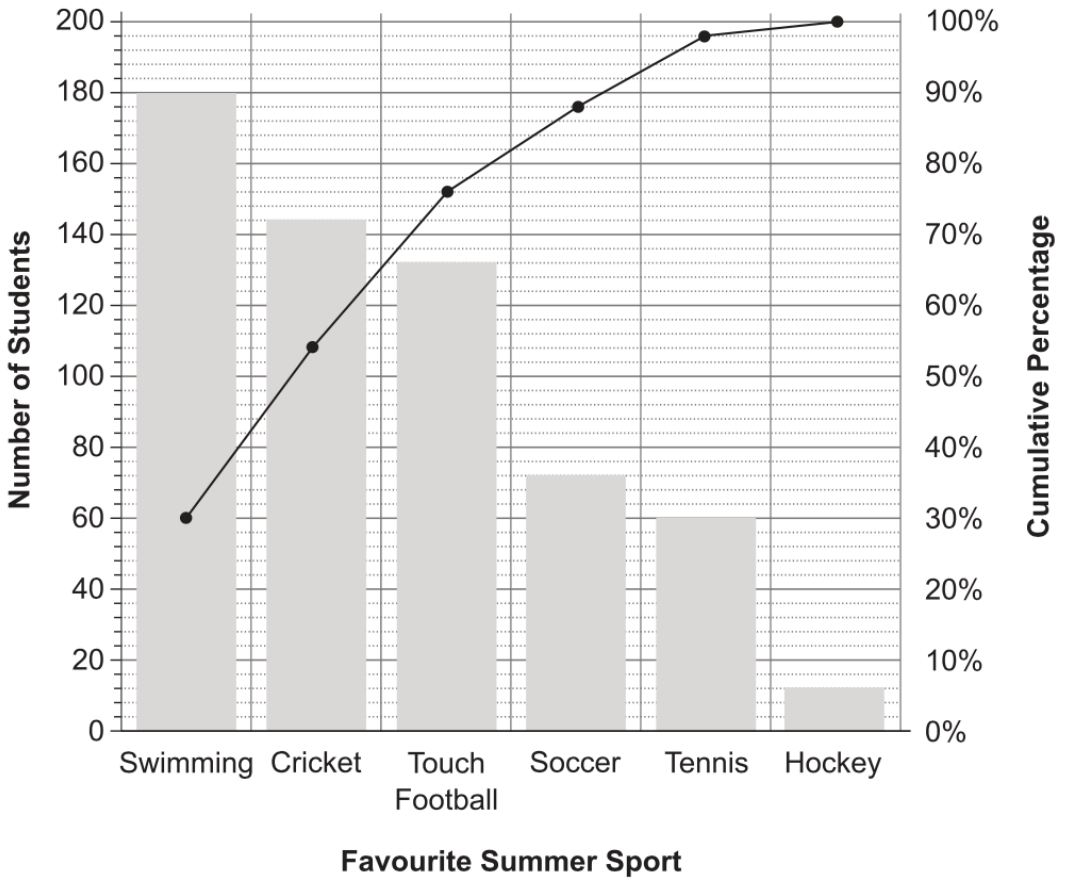
Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2022 Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

1 A high school conducted a survey asking students what their favourite Summer sport was. The Pareto chart shows the data collected.



What percentage of students chose soccer as their favourite sport?

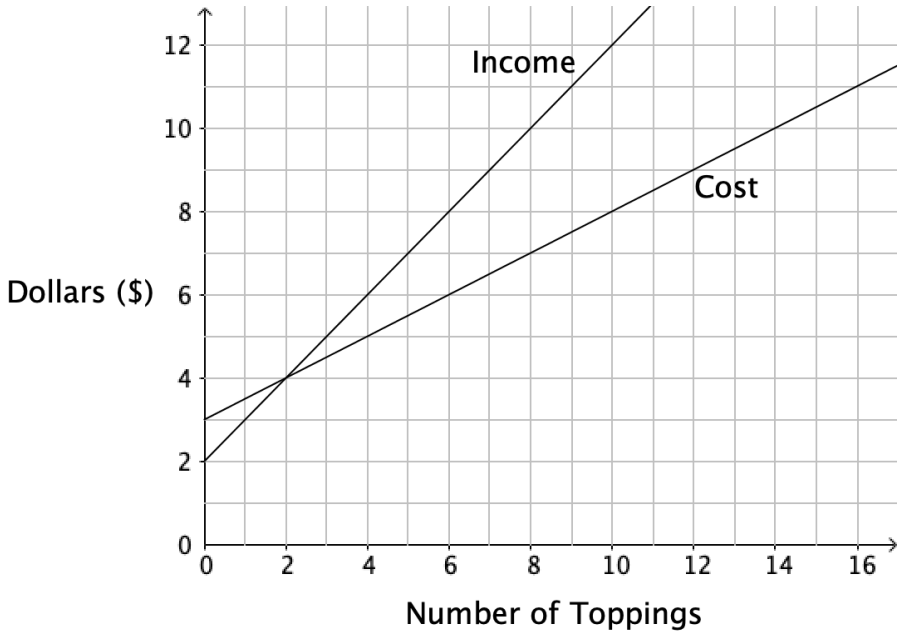
A. 10%

B. 12%

C. 36%

D. 72%

2 The graph below shows the cost of making pizzas depending on the number of toppings and the amount of income made from their sale.



How many toppings need to be sold to break even?

A. 0

B. 2

C. 5

D. 8

3 A jar of lollies contains 8 strawberry flavoured lollies and 9 apple flavoured lollies. Dianne takes two lollies at random. What is the probability that she takes two of the same flavour?

A. $\frac{1}{2}$

B. $\frac{9}{34}$

C. $\frac{8}{17}$

D. $\frac{145}{269}$

4 $y = f(x)$ is an odd function. The value of $\int_{-a}^{a} f(x)\,dx$ is:

A. $f(a)$

B. $2\int_{0}^{a} f(x)\,dx$

C. 0

D. a

5 The linear function $f(x) = 5 - x$ has range $[-4, 5)$. The domain of the function is

A. $(0,9]$

B. $(0, 1]$

C. $[5, -4)$

D. $(-9,0)$

6 The random variable, X, has a normal distribution with mean 12 and standard deviation 0.25. Which of the following corresponds to X is greater than 12.5?

A. $P(Z < -1.5)$

B. $P(Z < 1)$

C. $P(Z \geq 1.5)$

D. $P(Z > 2)$

7 The average rate of change of the function $f(x) = 3x^2 - 2\sqrt{x+1}$, between $x = 0$ and $x = 3$, is

A. 8

B. 25

C. $\frac{53}{9}$

D $\frac{25}{3}$

8 In a group of 60 students, 26 students study Mathematics Advanced, 29 students study Biology, and 14 study both. Find the probability that a student studies Biology given that they study Mathematics Advanced.

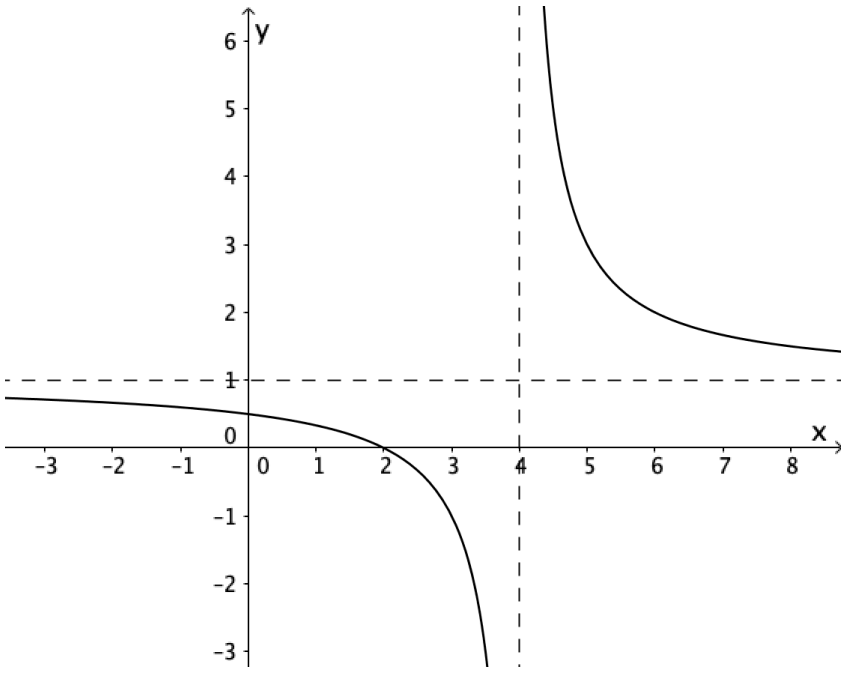
A. $\frac{7}{30}$

B. $\frac{7}{20}$

C. $\frac{7}{13}$

D. $\frac{29}{40}$

9 The graph of is $y = \frac{x-2}{x-4}$ shown below.



The values of x for which $\frac{x-2}{x-4} \leq 3$ are:

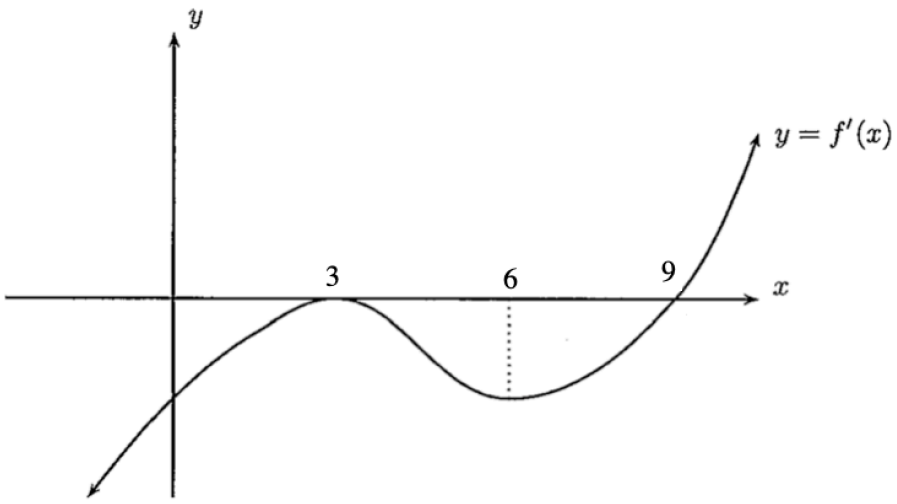
A. $x < 4, x \geq 5$

B. $x \leq 3$

C. $2 \leq x < 4$

D. $x < 1$

10 The diagram below shows the graph of the gradient function $y = f'(x)$ of the function $y = f(x)$. For what values of x is the function $y = f(x)$ increasing?



A. $x > 6$

B. $3 < x < 9$

C. $x < 3,\ x > 6$

D. $x > 9$

End of Section I

Student Name: _____

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

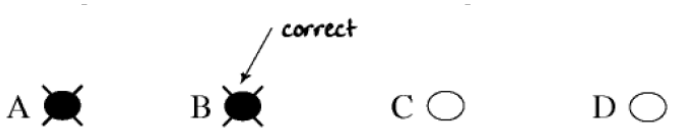
Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.



Start Here ➔

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

Mathematics Advanced

Section II Answer Booklet 1

90 marks
Attempt Questions 11 – 32
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11 – 23 (45 marks)
Booklet 2 – Attempt Questions 24 – 32 (45 marks)

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

NAME: _____

Question 11 (2 marks)

Evaluate $\cot\frac{4\pi}{3} \times \sin\frac{5\pi}{4}$. **2**

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Question 12 (2 marks)

Find the 19th term of the given geometric series **2**

$$2 - 6 + 18 - 54 + \ldots$$

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Question 13 (3 marks)

Find the sum of the given arithmetic series. **3**

$$9 + 16 + 23 + \ldots + 2963$$

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Question 14 (2 marks)

Let $f(x) = -x^2 + 2x - 3$ and $g(x) = x^2 + 3$. Express $f\big(g(x)\big)$ in the form of $ax^4 + bx^2 + c$, where a, b, and c are non-zero integers. **2**

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Question 15 (4 marks)

Differentiate the following functions and simplify your answers where necessary.

(a) $y = e^{5x}\cos 3x$ **2**

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(b) $y = \dfrac{10x - 3}{x^2 + 1}$ **2**

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Question 16 (4 marks)

Find

(a) $\int \left(\frac{2}{x} + e^{2x}\right) dx$ **2**

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(b) $\int \frac{1}{(5x-7)^3} dx$ **2**

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Question 17 (3 marks)

The curve $y = f(x)$ has gradient function $f'(x) = 3x^2 - k$, where k is a constant.

(a) Find the value of k if the curve has a stationary point at $(-1, 3)$. **1**

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(b) Hence, find the equation of the curve $y = f(x)$. **2**

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Question 18 (6 marks)

The discrete random variable X has this probability distribution.

X	0	1	2	3	4
$P(X = x)$	0.2	k	0.1	k	0.1

(a) Show that the value of k is 0.3. **1**

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(b) Calculate the expected value and variance of X. **2**

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(c) Find $P(X \leq 1)$ **1**

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(d) Find $P(X > 1 \mid X \leq 3)$ **2**

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Question 19 (4 marks)

The bank has approved Raj's loan of \$70000 for him to get a new car. The bank charges interest at 0.85% per month. Raj pays \$2000 at the beginning of every month to pay off his loan.

The amount in the account immediately after the *n*th repayment can be determined using the recurrence relation

$$A_n = A_{n-1}(1.0085) - 2000,$$

where $n = 1, 2, 3, \ldots$ and $A_0 = 70000$

(a) Use the recurrence relation to find the amount of money in the account immediately after the third repayment. **2**

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(b) Calculate the amount of interest added in the first three months. **2**

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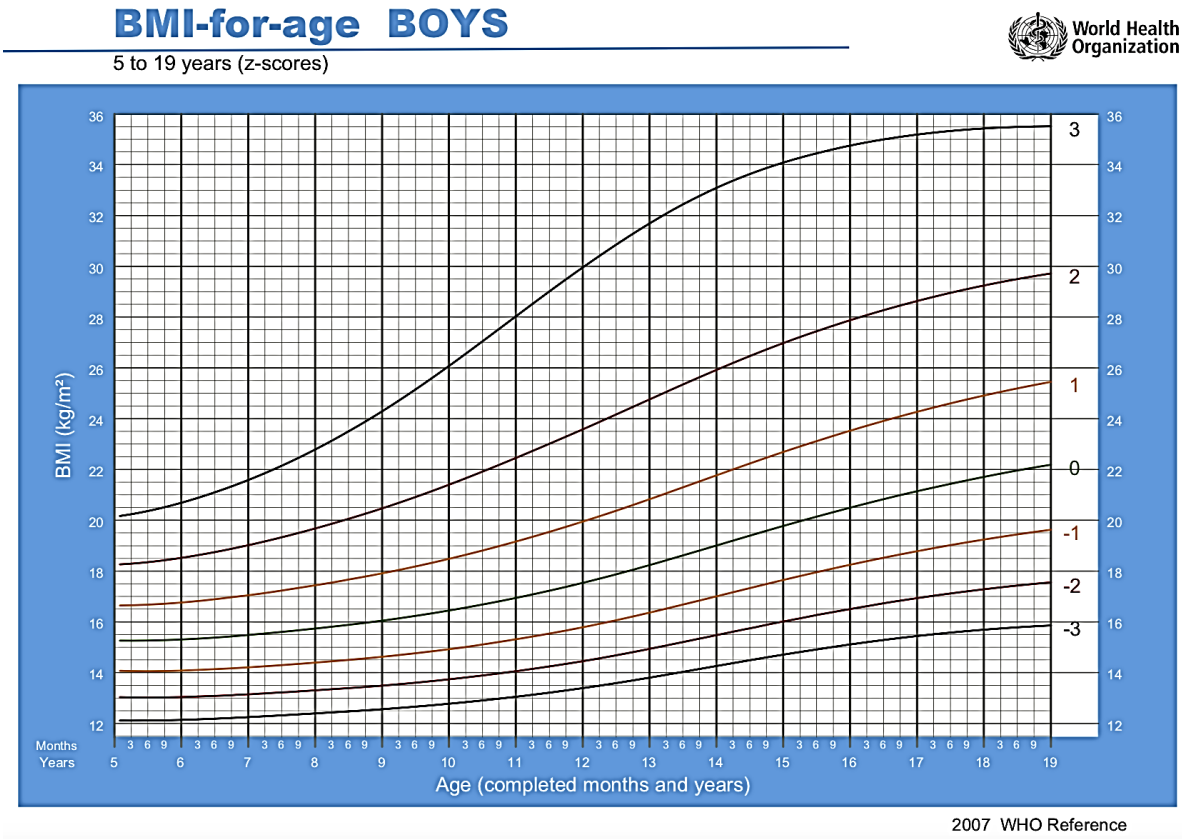
Question 20 (4 marks)

Body mass index (BMI) is a measure that can be used to determine a person's healthy weight range. The following graph indicates z-scores of 'BMI-for age' for boys aged 5-19 years.



(a) How old is Jackson if he has a BMI of 20 and a z-score of 1? **1**

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(b) Benjamin is 9 years old. If 97.5% of boys his age have a higher BMI than him, what is his BMI? **1**

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(c) George is 16 years old and has a BMI of 23.5. There are 40 people in his Year 10 cohort who have a BMI larger than his. Assuming all students are 16 years old, how many people are in George's cohort? **2**

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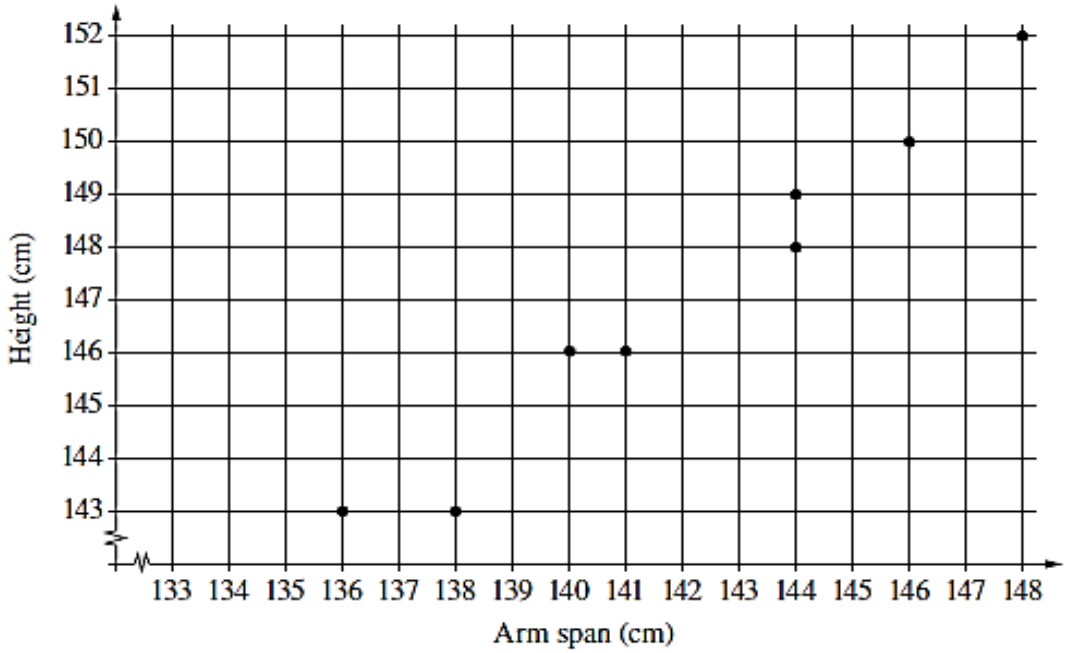
Question 21 (4 marks)

A set of data is collected by measuring the height and arm span of eight children. The graph shows a scatterplot of these measurements.



(a) Calculate the value of Pearson's correlation coefficient, correct to four decimal places. **1**

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(b) Hence describe the association between the height and arm span in terms of strength and direction. **1**

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(c) Determine the equation of the least-squares regression line for this data. Round your values to two significant figures. **2**

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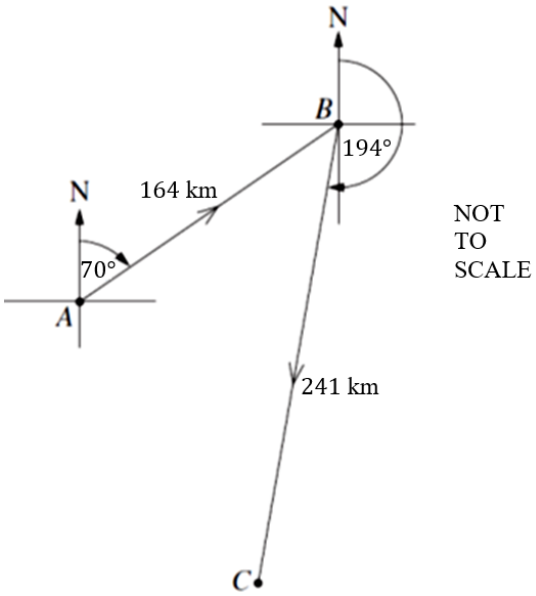
Question 22 (5 marks)

Manavdeep leaves island A in a boat and sails 164 km on a bearing of 070° to island B. He then sails on a bearing of 194° for 241 km to island C, as shown in the diagram.



(a) Show that the size of $\angle ABC = 56°$. **1**

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(b) Show that the distance from island C to island A is approximately $202\ km$. **2**

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(c) Manavdeep wants to sail from island C directly to island A. On what bearing should he sail? Give your answer to the nearest degree. **2**

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Question 23 (2 marks)

An object rolls 0.5 metres in the first second. Then each second after, it rolls by two thirds of the distance of its previous roll. How far will it roll altogether eventually? **2**

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End of Question 23

Proceed to Booklet 2

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If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 – Attempt Questions 24 – 32 (45 marks)

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

NAME: _____

Question 24 (6 marks)

Leonard and Howard start jobs at the beginning of the same year. Leonard's annual salary in the first year is \$53000 and increases by 3% at the beginning of each subsequent year. Howard's annual salary in the first year is \$59000 and increases by \$1500 at the beginning of each subsequent year.

(a) Show that in the 15^{th} year, Leonard's annual salary is higher than Howard's annual salary. **2**

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(b) In the first 15 years, how much in total, does Leonard earn? **2**

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(c) In which year will Leonard start to earn a salary more than \$90000 a year? **2**

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Question 25 (4 marks)

A probability density function $f(x)$ is given by

$$f(x) = \begin{cases} kx(5-x), & if\ 0 \le x \le 5 \\ 0, & if\ x < 0, x > 5 \end{cases}$$

where k is a positive constant.

(a) Show that $k = \frac{6}{125}$. **1**

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(b) Determine the mode of $f(x)$. **1**

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(c) Find $P(X > 3)$. **2**

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Question 26 (3 marks)

On any given day, the depth of water in a river is modelled by the function

$$h(t) = 14 + 8\sin\left(\frac{\pi t}{12}\right), 0 \leq t \leq 24$$

where h is the depth of water, in metres and t is the time, in hours, after 6 am.

(a) Find the minimum depth of the water in the river. **1**

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(b) Find the values of t for which $h(t) = 10$. **2**

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Question 27 (7 marks)

Consider the curve given by $y = x^3 - x^2 - x + 6$, for $-3 \leq x \leq 3$.

(a) Find the coordinates of the stationary points and determine their nature. **3**

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(b) Show that there is a point of inflection at $\left(\frac{1}{3}, \frac{151}{27}\right)$. **1**

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Question 27 continues on page 28

(c) Sketch the curve for $-3 \leq x \leq 3$, showing all key features. **2**

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(d) For what values of x is the curve decreasing and concaving up? **1**

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End of Question 27

Question 28 (4 marks)

A tank contains 50 litres of water. A tap at the base of the tank allows water to flow out at a rate proportional to the quantity of water still in the tank at that time, given by the equation $W = W_0 e^{-kt}$, where W is the amount of water in the tank and t is time in minutes. After 2 minutes, 10 litres of water has been released.

(a) Show that $k = -\frac{1}{2}\ln\frac{4}{5}$. **2**

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(b) How much water has been released from the tank after 10 minutes? Give your answer to the nearest litre. **2**

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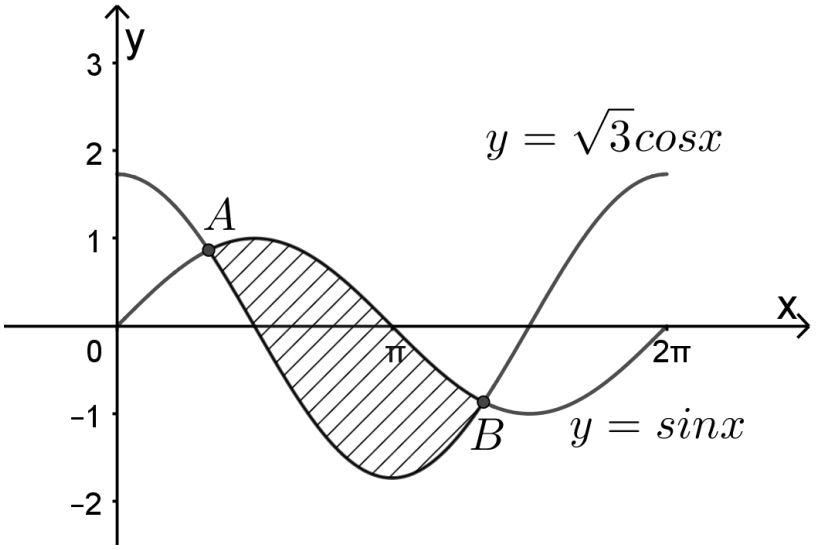
Question 29 (5 marks)

The diagram shows the graph of $y = \sin x$ and $y = \sqrt{3}\cos x$, $0 \leq x \leq 2\pi$. The graphs intersect at points A and B.



(a) Show that point A has coordinates $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$. **1**

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(b) Find the coordinates of point B. **1**

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Question 29 continues on page 31

Question 29 (continued)

(c) Find the shaded area enclosed by the two graphs. **3**

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End of Question 29

Question 30 (7 marks)

A particle is moving in a straight line, starting from the origin. At time t seconds, the particle has displacement x metres from the origin and velocity $v\ m/s$. The displacement is given by $x = 2t - 3\log_e(t+1)$.

(a) Find an expression for the velocity v. **1**

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(b) Find the initial acceleration. **2**

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(c) When does the particle come to rest? **2**

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Question 30 is continued on page 33

(d) Find the distance travelled by the particle in the first three seconds. Answer correct to four decimal places. **2**

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End of Question 30

Question 31 (4 marks)

(a) Show that $\frac{d}{dx}\left(\frac{1}{\sin x}\right) = -\cot x \operatorname{cosec} x$. **2**

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(b) Hence evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot x \operatorname{cosec} x \, dx$. Give your answer in exact form. **2**

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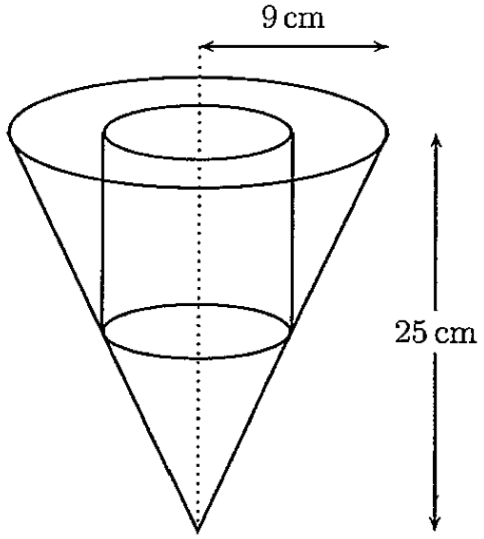
Question 32 (5 marks)

A cylinder is inscribed in a cone of radius 9 cm and perpendicular height 25 cm.



(a) Show that the perpendicular height h of the cylinder is given by **1**

$$h = \frac{25(9 - r)}{9},$$

where r is the radius of the cylinder.

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(b) Show that the volume of the cylinder is given by **1**

$$V = \frac{25\pi}{9}(9r^2 - r^3)$$

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Question 32 is continued on page 36

Question 32 (continued)

(c) Hence, find the maximum possible exact volume of this cylinder. **3**

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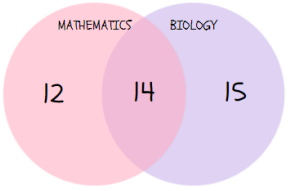
End of paper

2022 Year 12 Mathematics Advanced AT4 Solutions

Section 1

Q1	**B** $88\% - 76\% = 12\%$	1 Mark
Q2	**B** Break even is the point of intersection	1 Mark
Q3	**C** Two of the same flavours: two strawberries or two apples $P(SS) + P(AA) = \frac{8}{17} \times \frac{7}{16} + \frac{9}{17} \times \frac{8}{16} = \frac{8}{17}$	1 Mark
Q4	**C** $f(x)$ is an odd function $\int_{-a}^{a} f(x)\,dx = 0$	1 Mark
Q5	**A** $5 - x = -4 \quad 5 = 5 - x$ $x = 9 \quad x = 0$ Domain: $(0, 9]$	1 Mark
Q6	**D** $z = \frac{12.5 - 12}{0.25} = 2$ $P(X > 12.5) = P(Z > 2)$	1 Mark
Q7	**D** $f(0) = 3 \times 0^2 - 2 \times \sqrt{0 + 1} = -2$ $f(3) = 3 \times 3^2 - 2 \times \sqrt{3 + 1} = 23$ Average rate of change is $\frac{23 - (-2)}{3 - 0} = \frac{25}{3}$	1 Mark
Q8	**C**  Probability of a student that studies Biology given that they study Mathematics Advanced is $\frac{14}{26} = \frac{7}{13}$	1 Mark
Q9	**A** Draw a horizontal straight line of $y = 3$ on the original graph, look for sections of the original graph that is below the line $y = 3$. Intersection of the two graphs is where $x = 5$. $\therefore x < 4, x \geq 5$	1 Mark
Q10	**D** $f(x)$ is increasing if $f'(x) > 0$ $\therefore x > 9$	1 Mark

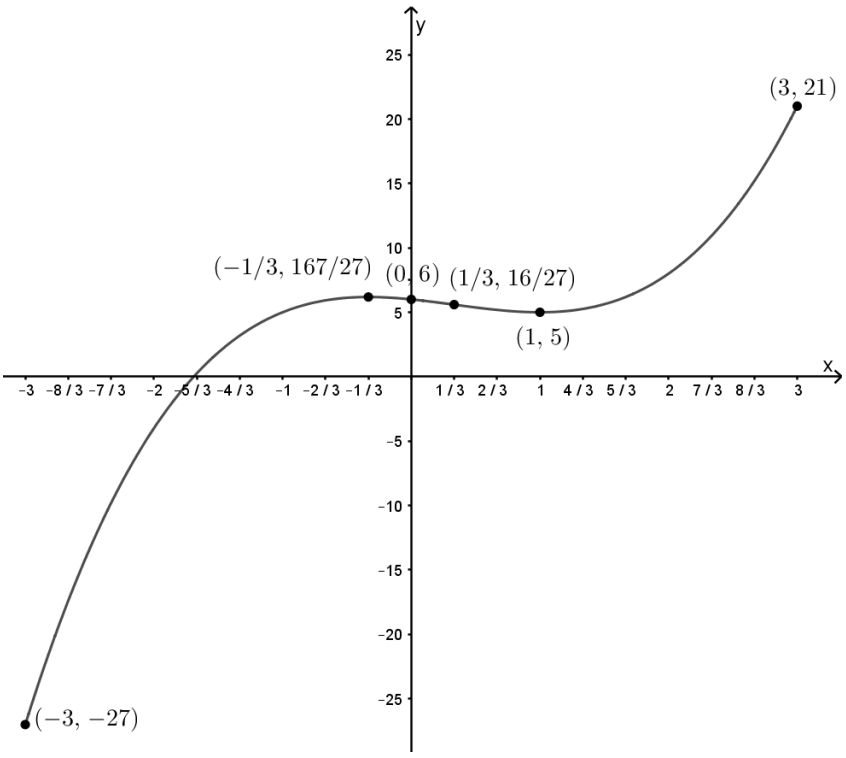
Section 2		
Q11	$\cot\frac{4\pi}{3} \times \sin\frac{5\pi}{4}$ $= \frac{1}{\tan\frac{4\pi}{3}} \times -\frac{1}{\sqrt{2}}$ $= \frac{1}{\sqrt{3}} \times -\frac{1}{\sqrt{2}}$ $= -\frac{1}{\sqrt{6}}$	2 Marks Correct solution 1 Mark Correct exact value for $\sin\frac{5\pi}{4}$ or $\cot\frac{4\pi}{3}$
Q12	$a = 2, r = -3$ $T_{19} = ar^{n-1}$ $T_{19} = 2 \times (-3)^{19-1}$ $T_{19} = 774840978$	2 Marks Correct solution 1 Mark Identifies a and r
Q13	$a = 9, d = 7$ $T_n = a + (n-1)d$ $2963 = 9 + (n-1) \times 7$ $n = 423$ $S_n = \frac{n}{2}(a + l)$ $S_n = \frac{423}{2}(9 + 2963)$ $S_n = 628578$	3 Marks Correct solution 2 Marks Finds $n = 423$ and attempts to find the sum 1 Mark Finds $2963 = 9 + (n-1) \times 7$
Q14	$f\big(g(x)\big) = f(x^2 + 3)$ $= -(x^2 + 3)^2 + 2(x^2 + 3) - 3$ $= -(x^4 + 6x^2 + 9) + 2x^2 + 6 - 3$ $= -x^4 - 4x^2 - 6$	2 Marks Correct solution 1 Mark Shows $f\big(g(x)\big)$ $= -(x^2 + 3)^2 + 2(x^2 + 3) - 3$
Q15a	$y = e^{5x}\cos 3x$ $\frac{dy}{dx} = e^{5x} \times -3\sin 3x + 5e^{5x} \times \cos 3x$ $\frac{dy}{dx} = -3e^{5x}\sin 3x + 5e^{5x}\cos 3x$	2 Marks Correct solution 1 Mark Correct differentiation of e^{5x} or $\cos 3x$ and attempts product rule
Q15b	$y = \frac{10x - 3}{x^2 + 1}$ $y = \frac{(x^2 + 1) \times 10 - (10x - 3) \times 2x}{(x^2 + 1)^2}$ $y = \frac{10x^2 + 10 - 20x^2 + 6x}{(x^2 + 1)^2}$ $y = \frac{-10x^2 + 6x + 10}{(x^2 + 1)^2}$	2 Marks Correct solution 1 Mark Correct quotient rule
Q16a	$\int \left(\frac{2}{x} + e^{2x}\right) dx$ $= 2\log_e\lvert x\rvert + \frac{1}{2}e^{2x} + C$	2 Marks Correct solution 1 Mark Correct integration of $\frac{2}{x}$ or e^{2x}

Q16b	$\int \frac{1}{(5x-7)^3}dx$ $= \int (5x-7)^{-3}dx$ $= \frac{(5x-7)^{-2}}{-2 \times 5} + C$ $= \frac{1}{-10(5x-7)^2} + C$	2 Marks Correct solution 1 Mark Demonstrates understanding of $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)}$
Q17a	Stationery point at $(-1, 3)$ $f'(x) = 3x^2 - k$ $0 = 3 \times (-1)^2 - k$ $k = 3$	1 Mark Correct solution
Q17b	$f(x) = \int (3x^2 - 3)dx$ $f(x) = x^3 - 3x + C$ $3 = (-1)^3 - 3 \times (-1) + C$ $C = 1$ $\therefore f(x) = x^3 - 3x + 1$	2 Marks Correct solution 1 Mark Finds $f(x) = x^3 - 3x + C$
Q18a	$0.2 + k + 0.1 + k + 0.1 = 1$ $2k = 0.6$ $k = 0.3$	1 Mark Correct solution
Q18b	$E(X) = 0 \times 0.2 + 1 \times 0.3 + 2 \times 0.1 + 3 \times 0.3 + 4 \times 0.1$ $E(X) = 1.8$ $E(X^2) = 0^2 \times 0.2 + 1^2 \times 0.3 + 2^2 \times 0.1 + 3^2 \times 0.3 + 4^2 \times 0.1$ $E(X^2) = 5$ $Var(X) = E(X^2) - [E(X)]^2$ $Var(X) = 5 - 1.8^2$ $Var(X) = 1.76$	2 Marks Correct solution 1 Mark Correct $E(X)$ or $Var(X)$
Q18c	$P(X \leq 1) = 0.2 + 0.3$ $P(X \leq 1) = 0.5$	1 Mark Correct solution
Q18d	$P(X > 1 \mid X \leq 3) = \frac{P(X > 1 \cap X \leq 3)}{P(X \leq 3)}$ $P(X > 1 \mid X \leq 3) = \frac{P(X = 2) + P(X = 3)}{1 - P(X = 4)}$ $P(X > 1 \mid X \leq 3) = \frac{0.1 + 0.3}{1 - 0.1}$ $P(X > 1 \mid X \leq 3) = \frac{4}{9}$	2 Marks Correct solution 1 Mark Shows $P(X > 1 \cap X \leq 3) = 0.4$
Q19a	$An = A_{n-1}(1.0085) - 2000$ $A_1 = A_0 \times 1.0085 - 2000$ $A_1 = 70000 \times 1.0085 - 2000 = 68595$ $A_2 = A_1 \times 1.0085 - 2000$ $A_2 = 68595 \times 1.0085 - 2000 = 67178.0575$ $A_3 = A_2 \times 1.0085 - 2000$ $A_3 = 67178.0575 \times 1.0085 - 2000 = 65749.0709 \ldots$ $A_3 \approx \$65749.07$	2 Marks Correct solution 1 Mark Finds the correct value of A_1

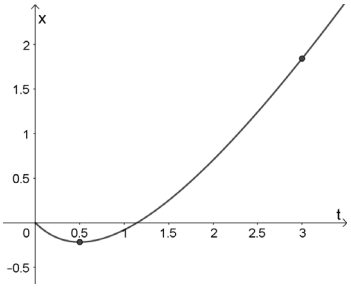
Q19b	Amount without interest in the first three months $= 70000 - 3 \times 2000 = 64000$ Amount with interest in the first three months $= \$65749.07$ Interest is $65749.07 - 64000 = \$1749.07$	2 Marks Correct solution 1 Mark Finds amount without interest in the first three months
Q20a	12 years old	1 Mark Correct solution
Q20b	$1 - 97.5\% = 2.5\%$ $z = -2$ $BMI = 13.5$	1 Mark Correct solution
Q20c	$z = 1$ $P(Z > 1) = \frac{1}{2}\big(1 - P(-1 \leq Z \leq 1)\big)$ $P(Z > 1) = \frac{1}{2}(1 - 68\%) = 16\%$ $16\% = 40$ people $100\% = 250$ people	2 Marks Correct solution 1 Mark Obtains $P(Z > 1) = 16\%$
Q21a	$r = 0.984682\ldots$ $r \approx 0.9847$	1 Mark Correct solution
Q21b	Positive and strong	1 Mark Correct solution
Q21c	$A = 36.617\ldots \quad B = 0.7775\ldots$ $A \approx 37 \quad B \approx 0.78$ Rounded to 2 significant figures $y = 0.78x + 37$	2 Marks Correct solution 1 Mark Correct A or B value
Q22a	$\angle ABC = 70° - (194° - 180°)$ $\angle ABC = 56°$	1 Mark Correct solution
Q22b	$AC^2 = 164^2 + 241^2 - 2 \times 164 \times 241 \times \cos 56°$ $AC = \sqrt{164^2 + 241^2 - 2 \times 164 \times 241 \times \cos 56°}$ $AC = 201.9255\ldots$ $AC \approx 202\ km$	2 Marks Correct solution 1 Mark Correct use of cos rule
Q22c	$\frac{\sin \angle ACB}{164} = \frac{\sin 56°}{202}$ $\sin \angle ACB = \frac{\sin 56°}{202} \times 164$ $\angle ACB = 42.305\ldots$ $\angle ACB = 42°$ (nearest degree) Bearing from C to A is $360° - [42° - (194° - 180°)] = 332°$	2 Marks Correct solution 1 Mark Correct value of $\angle ACB$
Q23	$a = 0.5, r = \frac{2}{3}$ $S_\infty = \frac{0.5}{1 - \frac{2}{3}}$ $S_\infty = 1.5$ metres	2 Marks Correct solution 1 Mark Correct substitution into the limiting sum formula

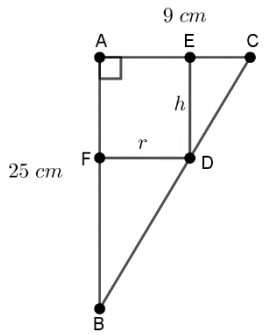
Q24a	Leonard $a = 53000, r = 1.03$ $L_1 = 53000,\ \ L_2 = 53000 \times 1.03,\ \ L_3 = 53000 \times 1.03^2, \ldots$ $L_{15} = 53000 \times 1.03^{14}$ $L_{15} = \$80167.25542 \ldots = \$80167.26$ Howard $a = 59000, d = 1500$ $H_1 = 59000,\ \ H_2 = 59000 + 1500,\ \ H_3 = 59000 + 1500 \times 2, \ldots$ $H_{15} = 59000 + 1500 \times 14$ $H_{15} = \$80000$ $L_{15} > H_{15}$	2 Marks Correct solution 1 Mark Correct value for Leonard or Howard
Q24b	$S_{15} = \dfrac{53000 \times (1.03^{15} - 1)}{1.03 - 1}$ $S_{15} = \$985742.435994 \ldots$ $S_{15} = \$985742.44$	2 Marks Correct solution 1 Mark Correct substitution into the sum formula
Q24c	$53000 \times 1.03^{n-1} > 90000$ $1.03^{n-1} > \dfrac{90}{53}$ $(n-1)\ln 1.03 > \ln\left(\dfrac{90}{53}\right)$ $n - 1 > \dfrac{\ln\left(\frac{90}{53}\right)}{\ln 1.03}$ $n > \dfrac{\ln\left(\frac{90}{53}\right)}{\ln 1.03} + 1$ $n > 18.91406 \ldots$ $n = 19$	2 Marks Correct solution 1 Mark Correct substitution into the sum formula
Q25a	$\int_0^5 kx(5-x)dx = 1$ $k\int_0^5 (5x - x^2)dx = 1$ $k\left[\dfrac{5x^2}{2} - \dfrac{x^3}{3}\right]_0^5 = 1$ $k \times \dfrac{125}{6} = 1$ $k = \dfrac{6}{125}$	1 Mark Correct solution
Q25b	The mode occurs at the maximum turning point of this concaving down parabola. The two x intercepts are $x = 0$ and $x = 5$. Vertex occurs at $x = \dfrac{0+5}{2} = 2.5$ $\therefore$ The mode of $f(x)$ is $x = 2.5$	1 Mark Correct solution

Q25c	$P(X > 3) = 1 - P(X \leq 3)$ $P(X > 3) = 1 - \frac{6}{125}\int_0^3 (5x - x^2)dx$ $P(X > 3) = 1 - \frac{6}{125}\left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_0^3$ $P(X > 3) = 1 - \frac{6}{125}\left[\frac{5 \times 3^2}{2} - \frac{3^3}{3} - 0\right]$ $P(X > 3) = 1 - \frac{6}{125} \times \frac{27}{2}$ $P(X > 3) = \frac{44}{125}$	2 Marks Correct solution 1 Mark Identifies $P(X > 3)$ $= 1 - \frac{6}{125}\int_0^3 (5x - x^2)dx$
Q26a	$h_{min} = 14 - 8$ $h_{min} = 6$ metres	1 Mark Correct solution
Q26b	$14 + 8\sin\left(\frac{\pi t}{12}\right) = 10$ $8\sin\left(\frac{\pi t}{12}\right) = -4$ $\sin\left(\frac{\pi t}{12}\right) = -\frac{1}{2}$ $\frac{\pi t}{12} = \pi + \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$ $\frac{\pi t}{12} = \frac{7\pi}{6},\ \frac{11\pi}{6}$ $t = 14, 22$	2 Marks Correct solution 1 Mark Finds $\frac{\pi t}{12} = \frac{7\pi}{6}, \frac{11\pi}{6}$ Or One correct value of t
Q27a	$y = x^3 - x^2 - x + 6$ $\frac{dy}{dx} = 3x^2 - 2x - 1$ $3x^2 - 2x - 1 = 0$ $(3x + 1)(x - 1) = 0$ $x = -\frac{1}{3},\ x = 1$ $y = \frac{167}{27},\ y = 5$ $\frac{d^2y}{dx^2} = 6x - 2$ $at\ x = -\frac{1}{3},\ \frac{d^2y}{dx^2} = 6 \times -\frac{1}{3} - 2 = -4 < 0$ $at\ x = 1,\ \frac{d^2y}{dx^2} = 6 \times 1 - 2 = 4 > 0$ $\therefore \left(-\frac{1}{3}, \frac{167}{27}\right)$ is a max turning point, $(1, 5)$ is a min turning point	3 Marks Correct solution 2 Marks Finds both stationery points 1 Mark Finds the correct x values for the turning points

Q27b	$\frac{d^2y}{dx^2} = 6x - 2$ $6x - 2 = 0$ $x = \frac{1}{3}$ $y = \left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right) + 6 = \frac{151}{27}$ x | 0 | $\frac{1}{3}$ | 1 $\frac{d^2y}{dx^2}$ | -2 | 0 | 4 There is a change in concavity, therefore the point of inflection is $\left(\frac{1}{3}, \frac{151}{27}\right)$	1 Mark Correct solution
Q27c	$x = -3, f(-3) = -27$ $x = 3, f(3) = 21$ y intercept, $f(0) = 6$  	2 Marks Correct solution 1 Mark Correct curve with some key features shown
Q27d	Decreasing and concaving up $\frac{1}{3} < x < 1$	1 Mark Correct solution
Q28a	$W_0 = 50$ $W = 50e^{-kt}$ $t = 2, W = 40$ $40 = 50e^{-2k}$ $\frac{4}{5} = e^{-2k}$ $\ln\frac{4}{5} = -2k$ $k = -\frac{1}{2}\ln\frac{4}{5}$	2 Marks Correct solution 1 Mark Shows $40 = 50e^{-2k}$

Q28b	$t = 10$ $W = 50e^{-10\times-\frac{1}{2}\ln\frac{4}{5}}$ $W = 16.384$ 16.384 litres of water are in the tank. $50 - 16.384 = 33.616$ $\therefore 34$ litres of water has been released.	2 Marks Correct solution 1 Mark Finds $W = 16.384$
Q29a	$A\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$ Sub the x value into $y = \sin x$ and $y = \sqrt{3}\cos x$ $y = \sin\frac{\pi}{3}$ $\quad$ $y = \sqrt{3}\cos\frac{\pi}{3}$ $y = \frac{\sqrt{3}}{2}$ $\quad$ $y = \frac{\sqrt{3}}{2}$ Same y value obtained, therefore A is the one of the points of intersection of these two functions.	1 Mark Correct solution
Q29b	$\sin x = \sqrt{3}\cos x$ $\tan x = \sqrt{3}$ $x = \frac{\pi}{3}, \ \pi + \frac{\pi}{3}$ $x = \frac{\pi}{3}, \ \frac{4\pi}{3}$ $y = \sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$ $B\left(\frac{4\pi}{3}, -\frac{\sqrt{3}}{2}\right)$	1 Mark Correct solution
Q29c	$A = \int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} (\sin x - \sqrt{3}\cos x)dx$ $A = \left[-\cos x - \sqrt{3}\sin x\right]_{\frac{\pi}{3}}^{\frac{4\pi}{3}}$ $A = \left(-\cos\frac{4\pi}{3} - \sqrt{3}\sin\frac{4\pi}{3}\right) - \left(-\cos\frac{\pi}{3} - \sqrt{3}\sin\frac{\pi}{3}\right)$ $A = \left(\frac{1}{2} + \sqrt{3}\times\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{2} - \sqrt{3}\times\frac{\sqrt{3}}{2}\right)$ $A = \left(\frac{1}{2} + 3\right) - \left(-\frac{1}{2} - 3\right)$ $A = 4\ units^2$	3 Marks Correct solution 2 Marks Correct primitive function 1 Mark Shows area is $\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} (\sin x - \sqrt{3}\cos x)dx$
Q30a	$x = 2t - 3\log_e(t+1)$ $v = 2 - \frac{3}{t+1}$	1 Mark Correct solution
Q30b	$v = 2 - 3(t+1)^{-1}$ $a = 3(t+1)^{-2}$ $a = \frac{3}{(t+1)^2}$ Initial acceleration when $t = 0$ $a = \frac{3}{(0+1)^2} = 3\ m/s^2$	2 Marks Correct solution 1 Mark Correct acceleration equation

Q30c	Particle at rest when $v = 0$ $2 - \frac{3}{t+1} = 0$ $\frac{3}{t+1} = 2$ $t + 1 = \frac{3}{2}$ $t = \frac{1}{2}\ s$	2 Marks Correct solution 1 Mark Shows $2 - \frac{3}{t+1} = 0$
Q30d	$t = 3$ $x = 2 \times 3 - 3\log_e(3+1)$ $x = 6 - 3\log_e 4$ $x = 1.841116\ldots$ $t = \frac{1}{2}$ $x = 2 \times \frac{1}{2} - 3\log_e\left(\frac{1}{2}+1\right)$ $x = 1 - 3\log_e \frac{3}{2}$ $x = -0.216395\ldots$  Total distance travelled is $(6 - 3\log_e 4) + 2\left\lvert 1 - 3\log_e \frac{3}{2}\right\rvert$ $= 2.273907\ldots$ $\approx 2.2739\ m$	2 Marks Correct solution 1 Mark Finds $x = 6 - 3\log_e 4$
Q31a	$\frac{d}{dx}\left(\frac{1}{\sin x}\right)$ $= \frac{d}{dx}(\sin x)^{-1}$ $= -1 \times \cos x \times (\sin x)^{-2}$ $= -\frac{\cos x}{\sin x} \times \frac{1}{\sin x}$ $= -\cot x \operatorname{cosec} x$	2 Marks Correct solution 1 Mark Correct differentiation
Q31b	$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot x \operatorname{cosec} x\ dx$ $= \left[-\frac{1}{\sin x}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ $= -\left[\frac{1}{\sin\frac{\pi}{2}} - \frac{1}{\sin\frac{\pi}{3}}\right]$ $= -\left[1 - \frac{2}{\sqrt{3}}\right]$ $= \frac{2}{\sqrt{3}} - 1$	2 Marks Correct solution 1 Mark Correct primitive function

Q32a	$\Delta ABC\|\|\|\Delta EDC$ (equiangular) $\frac{EC}{AC}=\frac{ED}{AB}$ $\frac{9-r}{9}=\frac{h}{25}$ $h=\frac{25(9-r)}{9}$  	1 Mark Correct solution
Q32b	$V=\pi r^2 h$ $V=\pi r^2\times\frac{25(9-r)}{9}$ $V=\frac{25\pi}{9}(9r^2-r^3)$	1 Mark Correct solution
Q32c	$\frac{dV}{dr}=\frac{25\pi}{9}(18r-3r^2)$ $\frac{25\pi}{9}(18r-3r^2)=0$ $3r(6-r)=0$ $r=0\ cm,\qquad r=6\ cm$ If $r=0\ cm$, the cylinder doesn't exist. $\frac{d^2V}{dr^2}=\frac{25\pi}{9}(18-6r)$ At $r=6\ cm$ $\frac{d^2V}{dr^2}=\frac{25\pi}{9}(18-6\times 6)=-50\pi<0$ $\therefore r=6$ produces maximum volume. Maximum volume is $V=\frac{25\pi}{9}(9\times 6^2-6^3)$ $V=300\pi\ cm^3$	3 Marks Correct solution 2 Marks Shows $r=6\ cm$ produces maximum volume 1 Mark Obtains $r=6\ cm$